# International Series in Analysis

## A Textbook in Modern Analysis

Editor

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# Analysis II

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# Preface

This is the second and last part of an introduction to analysis which is based on my beginner's course Analysis I–III and the more advanced course Tensoranalysis that I use to give in Heidelberg from time to time.

The present volume comprises material for a four semesters course. It starts with a fairly comprehensive introduction to functional analysis, which might serve as a basis for a separate independent lecture.

In the next two chapters the theory of differentiation in Banach spaces and the fundamental existence theorems in analysis are treated in great detail and generality.

In Chapter 9 we develop the existence and regularity theory for ordinary differential equations in Banach spaces always having in mind to apply these results later to differential equations of arbitrary order in semi-Riemannian manifolds.

The last three chapters, Chapter 10–12, contain some fairly advanced topics from measure theory and differential geometry. In addition to providing the basic definitions and results of these theories we included material that is of great importance from an analytical point of view like covering theorems, Hausdorff measures and vector valued measures, or a thorough treatment of submanifolds, tubular neighbourhoods, the Riemannian and Lorentzian distance functions with respect to a hypersurface, and solving evolution equations in manifolds.

The only topic that is missing, at least from my point of view, is the treatment of partial differential equations, especially looking at partial differential equations in manifolds. However, after reading the material in Chapter 11 and 12, anyone, who knows the PDE theory in Euclidean space, should be able to apply this theory to PDE problems in semi-Riemannian manifolds.

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