Graduate Series in Analysis

A Textbook in

Modern Analysis

IP International Press

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Analysis I

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Preface

This is the first part of an introduction to analysis in two volumes, an expanded version of the material of my courses Analysis I-III which I have given in Heidelberg over the years.

The mathematics taught in these courses ranged from elementary calculus to fairly advanced topics in functional analysis, measure theory and differential geometry. I have to admit, however, that I never managed to cover all the material, that will be presented in the two volumes, in three semesters; almost always a course in tensor analysis was offered as a continuation in the fourth semester.

After two years the students then had a solid knowledge in classical analysis, theory of differentiation in Banach spaces, measure theory and tensor analysis in semi-Riemannian manifolds enabling them to study more advanced topics in mathematics as well as physics.

The present textbook comprises the material for a one and a half semester course. After some fundamental concepts of logic, set theory and the real numbers have been introduced, the actual analysis starts in Chapter 1. The convergence of sequences and series is first examined in the real axis, then generalized to $\mathbb{R}^n$, and is finally treated, in great detail, in metric spaces resp. Banach spaces.

In the next chapters, topological concepts—continuity, compactness, connectedness—(Chapter 2) resp. differentiation in one variable (Chapter 3) are presented.

Chapter 4 contains the important theorems of Arzelà–Ascoli and Stone-Weierstraß as well as a section on analytic functions in several variables.

The last chapter deals with the Riemann integral. It is introduced for Banach space valued functions—the definitions and proofs are the same as in the real valued case, and the integration theory for regulated functions, which is normally used to deal with vector valued functions, is avoided.

This textbook is intended for first year graduate students or for undergraduates who later want to graduate in mathematics or physics. The formal prerequisites to understand the book are very low, and are certainly fulfilled by anyone who has passed high school successfully, but the learning curve might be a little bit too steep for those who do not want to graduate in mathematics or physics.

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The present volume is a translation of the German edition.

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