Übungen zur Funktionalanalysis

Blatt 2

1 Let E, F be Banach spaces, $\mathit{\Gamma} \subset E$ an open connected cone, and $\varphi \in C^1(\mathit{\Gamma}, F)$. Then φ is positively homogeneous of degree $k \in \mathbb{R}$ if and only if

$$D\varphi(x)x = k\varphi(x) \qquad \forall\, x \in \mathit{\Gamma}.$$

If you can't prove it for arbitrary Banach spaces you may assume that the spaces are finite dimensional. $\boxed{6}$

2 Prove that any vector space E has an algebraic basis, a so-called Hamel basis. $\boxed{2}$

3 Let E be an infinite dimensional Banach space, then its algebraic dimension is uncountable. $\boxed{2}$

4 Let E be a normed space, $\mathit{\Lambda} \subset E$ an a.c. (at most countable) complete subset, then $\mathit{\Lambda}$ contains an a.c. basis M. $\boxed{2}$

5 Prove directly that the l_p-spaces are complete, i.e., without using the fact that all of them are dual spaces. $\boxed{2}$