A CORRECTION ON "HYPERSURFACES OF PRESCRIBED WEINGARTEN CURVATURE"

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The proof of Lemma 1.5 in [1] in which we state that the inverses of the symmetric polynomials belong to the class of curvature functions to which the results of that paper apply is incorrect, as was pointed out to us by Thomas Nehring.

To fix the proof of the lemma replace formula (1.23) and the rest of the proof in [1] by the following:

$$\tag{1.23}\begin{aligned}\frac{\partial^2 F}{\partial\kappa_i\,\partial\kappa_j} = (k+1)F^{-1}F_iF_j - \frac{1}{k}\Big(\sum_{\alpha\in I}\kappa^{-\alpha}\Big)^{-\frac{1}{k}-1}\sum_{\alpha\in I}\kappa^{-\alpha_{ij}}\alpha(i)\alpha_i(j)\kappa_i^{-2}\kappa_j^{-2}\\ -\frac{2}{k}\Big(\sum_{\alpha\in I}\kappa^{-\alpha}\Big)^{-\frac{1}{k}-1}\sum_{\alpha\in I}\kappa^{-\alpha_i}\alpha(i)\kappa_i^{-3}\delta_{ij}\,.\end{aligned}$$

Let $\xi\in\mathbf{R}^n$, then, the corresponding quadratic form can be expressed as the sum of three terms

$$\tag{1.23a}\frac{\partial^2 F}{\partial\kappa_i\,\partial\kappa_j}\xi^i\xi^j = I_1+I_2+I_3\,,$$

where the last two are of special interest to us. In the second term we can replace $\alpha_i(j)$ by $\alpha(j)$ if we only sum over $i\neq j$. Furthermore, we observe that α_{ij} is symmetric and that $\kappa^{-\alpha_{ij}}\alpha(j)\kappa_j^{-1}=\kappa^{-\alpha_i}\alpha(j)$ with the corresponding relation when i and j are exchanged. Since $\alpha(i)$ is either 0 or 1 and the κ_i are positive we conclude

(1.23b)

$$I_2 = -\frac{1}{k}\Big(\sum_{\alpha\in I}\kappa^{-\alpha}\Big)^{-\frac{1}{k}-1}\sum_{\alpha\in I}\sum_{i\neq j}(\kappa^{-\alpha_i})^{1/2}\alpha(i)\kappa_i^{-3/2}(\kappa^{-\alpha_j})^{1/2}\alpha(j)\kappa_j^{-3/2}\xi^i\xi^j\,.$$

Evidently, I_3 is exactly twice the missing diagonal term in I_2 and we infer

$$\tag{1.23c}\frac{\partial^2 F}{\partial\kappa_i\,\partial\kappa_j}\xi^i\xi^j \le (k+1)F^{-1}(F_i\xi^i)^2 - \sum_i(F_i\xi^i)(\kappa_i^{-1}\xi^i)\,.$$

Now, let us choose a coordinate system such that $h_{ij}=\kappa_i\delta_{ij}$ and let (η_{ij}) be diagonal, then we conclude from (1.7) and the preceding estimate

$$\tag{1.24}F^{ij,kl}\eta_{ij}\eta_{kl}\le(k+1)F^{-1}(F^{ij}\eta_{ij})^2 - F^{ik}\tilde h^{jl}\eta_{ij}\eta_{kl}\,,$$

i.e. the $\tilde\sigma_k$ are of class (K).

References

1. Claus Gerhardt, *Hypersurfaces of prescribed Weingarten curvature*, Math. Z. **224** (1997), 167–194.

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