Übungen zu Analysis III

Blatt 9

1 Let $f \in C^1(\Omega, E)$ be a time independent vector field, and let $x_1 \in C^1(J_1, \Omega)$, $x_2 \in C^1(J_2, \Omega)$ be two maximally defined integral curves of f that intersect, then there exists $\tau \in \mathbb{R}$ such that $J_1 = J_2 + \tau$ and

$$x_1(t + \tau) = x_2(t) \qquad \forall\, t \in J_2.$$

$\boxed{2}$

2 Let $\varphi \in C^1([0, b))$ be a solution of the linear differential equation

$$\dot{\varphi} = a\varphi$$

with $a \in C^0([0, b))$, then $\varphi \equiv 0$, if $\varphi(0) = 0$, $\varphi > 0$, if $\varphi(0) > 0$, and $\varphi(0) < 0$, if $\varphi(0) < 0$. $\boxed{3}$

3 Let $J = [0, b)$, $a \in C^0(J)$ and assume that $\varphi \in C^1(J)$ satisfies the differential inequality

$$\dot{\varphi} \ge a\varphi,$$

then $\varphi \ge 0$, if $\varphi(0) \ge 0$. $\boxed{2}$

Note: Consider the function $\tilde{\varphi} = \varphi e^{\lambda t}$ with $\lambda \in \mathbb{R}$ chosen appropriately.

4 Let H be a real Hilbert space, $f \in C^1(H, H)$ and suppose

$$\langle f(x), x \rangle \ge c\, \|x\|^{2+\epsilon} \qquad \forall\, x \in H$$

with positive constants c, ϵ. Let $x = x(t)$ be an integral curve of f with initial value x_0 and maximal domain of definition $J = (a, b)$, then b is a priori bounded, $b \le \text{const}(x_0, c, \epsilon)$, unless $x_0 = 0$ and $f(0) = 0$. $\boxed{4}$