Übungen zu Analysis III

Blatt 5

1 Suppose that the integral kernel $k = k(t,x)$ in Number 8.1.4 is uniformly Hölder continuous with respect to the variable x with exponent α, then the integral operator K in (8.1.12) is compact. $\boxed{4}$

2 Define polar coordinates in $\mathbb{R}^3$ $f(r,t,\theta) = (x,y,z)$ by setting

$$x = r\cos t \sin\theta, \quad y = r \sin t \sin\theta, \quad z = r\cos\theta,$$

where $(r,t,\theta) \in U = \mathbb{R}_+^* \times (0,2\pi) \times (0,\pi)$ and show that $f \in \operatorname{Diff}^\infty(U,V)$, where $V = f(U) = \mathbb{R}^3 \backslash H$ and H is the closed half-plane $\mathbb{R}_+ \times \{0\} \times \mathbb{R}$. $\boxed{4}$

3 Let (ξ_i) resp. (ξ^i) be the components of a covariant resp. contravariant vector in a Euclidean coordinate system in $\mathbb{R}^2$. How are these components transformed by switching to polar coordinates (r,t), cf. Remark 7.4.12 on page 79. $\boxed{2}$

4 Prove Corollary 8.2.5. $\boxed{4}$

5 The trajectory of a particle in $\mathbb{R}^n$ satisfies, in Euclidean coordinates, the equation

$$\dot{x}^i(\tau) = f^i(\tau), \quad x(0) = x_0,$$

where the parameter τ represents the time. How does this equation look in a diffeomorphic coordinate system $\tilde{x} = \tilde{x}(x)$. $\boxed{2}$

6 Prove Lemma 8.2.3. $\boxed{8}$