Übungen zu Analysis III

Blatt 4

1 Let E be a Banach space, H a Hilbert space, and $f : \Omega \subset E \to H$ a map of class C^m, $m \in \mathbb{N}$, such that

$$f(x) \neq 0 \qquad \forall\, x \in \Omega.$$

Let $P(x)$ be the projection onto the subspace $\langle f(x) \rangle$, then the map

$$\begin{aligned} P : \Omega &\to L(H, H) \\ x &\to P(x) \end{aligned}$$

is also of class C^m. $\boxed{4}$

2 Let E be a Banach space, $\Omega \subset E$ open, and $u_i \in C^m(\Omega, \mathbb{R}^n)$, $1 \leq i \leq n$, such that the n-bein $(u_i(x))$ is a basis of $\mathbb{R}^n$ for all $x \in \Omega$. Consider $f \in C^m(\Omega, \mathbb{R}^n)$, then f can be expressed in the form

$$f = f^i u_i$$

with real valued functions $f^i : \Omega \to \mathbb{R}$. Prove that $f^i \in C^m(\Omega)$. $\boxed{8}$
Note: Exercise 1 is of little help in this situation.

3 Let E, F be Banach spaces, $\Omega \subset E$ open and $I = [a, b]$ an interval. Assume that the partial derivative of $f \in C^0(I \times \Omega, F)$ with respect to $x \in \Omega$ exists such that $D_2 f \in C^m(I \times \Omega, L(E, F))$, then

$$\Phi(x) = \int_a^b f(t, x)$$

is of class C^{m+1} and

$$D^k \Phi(x) = \int_a^b D_2^k f(t, x) \qquad \forall\, 1 \leq k \leq m + 1,$$

cf. Theorem 5.9.3. $\boxed{6}$