Übungen zu Analysis III

Blatt 2

1 Find a new proof of the former result that the function $\Phi(t) = e^{tA}$, where $A \in L(E)$ and $t \in \mathbb{R}$, is continuously differentiable and that $\Phi'(t) = Ae^{tA} = e^{tA}A$, cf. the proof of Theorem 3.5.3 of Analysis I.

2 Give a new proof for exercise 4 of Exercises 3.5.5 of Analysis I, which we now formulate as: Let $\Omega \subset \mathbb{K}$ be open and $A : \Omega \to GL(E)$ a differentiable mapping, then the map $B(x) = A(x)^{-1}$ is also differentiable and

$$B'(x) = -A(x)^{-1}A'(x)A(x)^{-1}.$$

3 Let $a_i(t) \in \mathbb{R}^n$, $1 \le i \le n$, be differentiable vector fields depending on a real parameter t, then $\varphi(t) = \det(a_1, \dots, a_n)$ is differentiable and

$$\dot{\varphi} = \sum_{i=1}^{n} \det(a_1, \dots, a_{i-1}, \dot{a}_i, a_{i+1}, \dots, a_n).$$

4 Let (g_{ij}) be a symmetric differentiable matrix in $\mathbb{R}^n$, the coefficients of which depend differentiably on a real parameter t, and set $(g^{ij}) = (g_{ij})^{-1}$. Then $g = \det g_{ij}$ and g^{ij} are differentiable and there holds

(i) $\dot{g} = g\, g^{ij}\dot{g}_{ij}$,

(ii) $\dot{g}^{ij} = -g^{ik}\dot{g}_{kl}g^{lj}$.

Notice that we use Einstein's summation convention to sum over repeated indices, where one of the indices is an upper (contravariant) index and the other a lower (covariant) index.

5 Let $x = x(\tilde{x})$ be a coordinate transformation in $\mathbb{R}^n$, as described in Remark 7.4.12, then

$$\delta^i_j = \frac{\partial \tilde{x}^i}{\partial x^k}\frac{\partial x^k}{\partial \tilde{x}^j}.$$