Übungen zu Analysis III

Blatt 13

1 Let $(E, \mathcal{M}, \mu)$ be a measure space and $f \in L^p(E)$, $1 \le p < \infty$, then f is bounded a.e.

2 Complete the proof of Theorem 10.6.1 by showing that the relation

$$\mu(O(\Omega)) = \mu(\Omega), \tag{0.1}$$

where O is an orthogonal transformation, also holds for open sets Ω with $\mu(\Omega) = \infty$ and not only for those with finite measure.

3 Prove Lemma 10.6.2.

4 Prove the relation (10.6.33) by induction with respect to n.

5 Let $A \in L(\mathbb{R}^n, \mathbb{R}^n)$ be invertible, then there exists an orthogonal transformation O and a selfadjoint, positive definite operator B such that

$$A = OB, \tag{0.2}$$

and hence, there exist orthogonal operators O_1, O_2 and diagonal matrix D such that

$$A = O_1 D O_2. \tag{0.3}$$

Hint: Prove first that there exist a positive definite operator B such that $A^*A = BB$.

6 Prove Corollary 10.6.5.

7 Complete the second part of the proof of Theorem 10.6.8.

8 Prove Corollary 10.6.9.

9 Let E be a vector space and $\emptyset \ne A \subset E$, then we define the *convex hull* of A, in symbols, $\langle A \rangle$, as the set of all finite convex combinations of vectors in A

$$\langle A \rangle = \{ \sum \lambda^i x_i \colon x_i \in A \ \wedge \ 0 \le \lambda^i \le 1, \ \sum \lambda^i = 1 \}. \tag{0.4}$$

Let E be a normed space, prove that

$$\operatorname{diam}\langle A \rangle \le \operatorname{diam} A. \tag{0.5}$$