Übungen zu Analysis III

Blatt 11

1 Let J be an open interval containing 0 and let $f, g \in C^0(J)$. Determine the general solution of the differential equation

$$\dot{x} = f(t)x + g(t). \tag{0.1}$$

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2 Find the general solutions of the following differential equations

$$t\dot{x} + 2x = 0, \tag{0.2}$$

$$(1 - t^2)\dot{x} - x = 0, \tag{0.3}$$

$$(1 - t^2)\dot{x} - x = 0, \tag{0.4}$$

$$\dot{x}\sin t + x\cos t = 0, \tag{0.5}$$

$$(3t^2 + 1)\dot{x} - 2tx = 6t, \tag{0.6}$$

$$(t^2 + 1)\dot{x} - (1 - t)^2 x = te^{-t}. \tag{0.7}$$

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3 Let $a_i \in C^0(J)$, $i = 0, 1, 2$, and let x_1 be a special solution of the so-called *Riccati's differential equation*

$$\dot{x} + a_2(t)x^2 + a_1(t)x + a_0 = 0. \tag{0.8}$$

Define a new vector field y by the transformation $x = x_1 + y^{-1}$ and solve the resulting differential equation for $y = y(t)$ and prove that all solutions of Riccati's equation are known, if a special solutions is known. 4

4 Apply this method to the following Riccati differential equations

$$\dot{x} - tx^2 + (2t - 1)x = t - 1; \quad \text{special solution} \quad x_1 = 1, \tag{0.9}$$

$$\dot{x} + tx^2 - 2t^2 x + t^3 = t + 1; \quad \text{special solution} \quad x_1 = t - 1, \tag{0.10}$$

$$\dot{x} + x^2 - (1 + 2e^t)x + e^{2t} = 0; \quad \text{special solution} \quad x_1 = e^t, \tag{0.11}$$

$$\dot{x} + x^2 + 3x + 2 = 0, \tag{0.12}$$

$$\dot{x} + 4x^2 - 9 = 0. \tag{0.13}$$

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