Übungen zu Analysis III

Blatt 10

1 Prove the claim (9.4.48). $\boxed{3}$

2 Prove Lemma 9.5.10. $\boxed{3}$

3 Let E, F be Banach spaces, $\Omega \subset E$, $U \subset F$ open, J an open interval containing 0 and $f \in C^0(J \times \Omega, E)$ satisfying $D_2^k f \in C^0(J \times \Omega)$, $1 \le k \le m$, and $D_2^m f$ is locally in J uniformly Hölder continuous with respect to $x \in \Omega$ with exponent $0 < \alpha \le 1$. Let $\varphi \in C^{m,\alpha}(U, \Omega)$, and assume that the initial value problem

$$\begin{aligned} \dot{x} &= f(t, x) \\ x(0) &= \varphi(\eta_0) \end{aligned}$$

has a solution in a compact interval $I = [a, b]$, $a < 0 < b$, such that $x \in C^1(I, E)$, then there exists an open interval J_0 and $\rho > 0$ such that $I \subset J_0 \subset J$, $B_\rho(\eta_0) \subset U$ and the flow $x = x(t, \eta)$ exists for all $(t, \eta) \in J_0 \times B_\rho(\eta_0)$ solving

$$\begin{aligned} \dot{x} &= f(t, x) \\ x(0, \eta) &= \varphi(\eta) \end{aligned}$$

and $D_2^k x \in C^{0,\alpha}(J_0 \times B_\rho(\eta_0))$, $0 \le k \le m$.

Hint: I know of two proofs by induction. Both use already known facts, some of which have first to be checked that they are still valid under the present assumptions. $\boxed{10}$