Übungen zu Analysis III

Blatt 1

1 Prove Remark 7.1.6.
2 Prove that in a complex Hilbert space, the function f in Example 7.1.9 is only differentiable in the origin.
3 Let E be a Banach space and $f : L(E) \to L(E)$ be defined by $f(A) = A^2$. Prove that f is differentiable and determine its derivative.
Note: The relation $f'(A)B = 2A \circ B$ is not valid.
4 Prove part (ii) of Proposition 7.1.14.
5 Let $\alpha \in \mathbb{N}^n$ be a multiindex, then $\varphi(x) = x^\alpha$ is continuously differentiable in $\mathbb{R}^n$.
6 Let E, F be Banach spaces, and $\Omega \subset E$ open and relatively compact—E is then necessarily finite dimensional–, then $C^1(\bar{\Omega}, F)$ is a Banach space.