Übungen zu Analyis II

Blatt 12

1 Prove that $L(E_1, \ldots, E_n; F)$ is a Banach space, if F is complete.

2 Prove that the isomorphism between $L(E, L_{k-1}(E,F))$ and $L_k(E,F)$ is norm preserving for all $k \geq 2$.

3 Let E be a complex normed space and $\varphi \in E^*$. Prove the identity

$$\varphi(x) = \operatorname{Re}\varphi(x) - \operatorname{Re}\varphi(ix) \qquad \forall\, x \in E$$

4 Consider in the real Hilbert space l_2 the basis elements e_i the i-th components of which are 1 while all other components are 0 and prove

$$e_i \rightharpoonup 0,$$

but the e_i do not converge in the norm topology.